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Optimal real time pricing in an agent-based retail market using a comprehensive demand response model

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ABSTRACT

In this paper, a weighted combination of different demand vs. price functions referred to as Composite Demand Function (CDF) is introduced in order to represent the demand model of consuming sectors which comprise different clusters of customers with divergent load profiles and energy use habitudes. Derived from the mathematical representations of demand, dynamic price elasticities are proposed to demonstrate the customers' demand sensitivity with respect to the hourly price. Based on the proposed CDF and dynamic elasticities, a comprehensive demand response (CDR) model is developed in this paper for the purpose of representing customer response to time-based and incentive-based demand response (DR) programs. The above model helps a Retail Energy Provider (REP) agent in an agent-based retail environment to offer day-ahead real time prices to its customers. The most beneficial real time prices are determined through an economically optimized manner represented by REP agent's learning capability based on the principles of Q-learning method incorporating different aspects of the problem such as price caps and customer response to real time pricing as a time-based demand response program represented by the CDR model. Numerical studies are conducted based on New England day-ahead market's data to investigate the performance of the proposed model.

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1. Introduction

1.1. Motivation and technique

Smart grids lean on bidirectional interactions between energy suppliers and different clusters of customers. The smart power grid follows demand response (DR) programs which produce responsive demand and result in valuable benefits such as better capacity factors for existing capacities, significant reliability, mitigation of market power and lower electricity prices for consumers [1,2]. Customers in smart grids intelligently adjust their load profiles according to some factors like the resultant benefit for the electricity use, the varying price of energy and the incentives offered by DR providers for load reduction.

In order to represent the hourly energy consumption of customers with divergent energy use habitudes and load profiles, a Composite Demand Function (CDF) is introduced in this paper which includes different demand vs. price functions such as linear, exponential, potential and logarithmic demand functions and is able to associate with other mathematical representations of demand. The CDF simulates customers' hourly demand as a function of hourly electricity prices. These customers involve in their preferable DR program based on their expected benefit for the use of electricity and the features of the offered DR programs. Each mathematical representation of demand corresponds to an hourly benefit function which demonstrates the expected benefit for the use of energy based on the hourly price of electricity, changes in customer demand and the price elasticity of demand. Here, a comprehensive demand response (CDR) model is developed in order to represent the hourly changes in the consumer response according to the expected benefit and the price elasticity of demand as well as hourly energy prices, offered incentives and predetermined penalties in different DR programs. Furthermore, instead of using fixed price elasticities, dynamic price elasticities are derived here based on the main definition of self elasticity. Dynamic price elasticities are employed in organizing the CDR model.

This model is applied here in simulating day-ahead Real Time Pricing (RTP) as a time-based DR program in an electricity retail market. The competitive retail market composed of intelligent, independent and communicative players like Distribution System Operator (DSO), Distributed Generators (DGs), Retail Energy

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Nomenclature

The main notations used throughout the paper are stated below for quick reference. Other symbols are defined throughout the text as required.

a, b	Coefficients of demand vs. price functions
$B^{C}(d(h))$	Customer's obtained benefit from consuming the hourly demand of $d(h)$ ($)
$B^{REP}(p(h))$	Retail energy provider's obtained benefit from selling energy at hourly price of $p(h)$ ($)
$d(h)$	Customer's hourly demand as a function of the hourly price (MWh)
$DR(h)$	Customer's hourly demand in response to a DR program (MWh)
$E(h)$	Dynamic price elasticity of demand
$inc(h)$	Hourly incentive per MWh of reduced load ($/MWh)
$p(h)$	Hourly price of electricity offered to the customers ($/MWh)
$pen(h)$	Hourly penalty per MWh of not reducing load within the contract level ($/MWh)

A symbol $\mathcal{F}(\forall \mathcal{F} \in \{lin, ptn, log, exp\})$ affecting any of the above notations indicates its value related to the function $\mathcal{F}$ representing any of linear, potential, logarithmic and exponential functions. Also, initial values are denoted by zero subscript.

Providers (REPs), and different clusters of customers is totally compatible with a Multi-Agent System (MAS) including some rational learner agents with cooperation, competition and negotiation interactions. Here, a complete structure of an agent-based retail energy market is presented and the problem of RTP for a customer agent is modeled based on the learning capability of an intelligent REP agent. REP agent's intelligence appears in the ability of learning the optimal pricing strategy by experiencing its impact on demands and consequent retail profit. The CDR model helps the REP agent to simulate the behavior of RTP program participants. Hereinafter, these customers are referred to as active customer agent. The learning process is modeled using Q-learning (QL) approach. In other word, optimum real time prices are determined through optimizing REP agent's benefit which is conducted using an optimization technique extracted from QL method. The acquired hourly benefits can be compared with the benefits resulted from real time pricing using a heuristic optimization technique like genetic algorithm (GA). It is assumed that the preliminary energy demand levels for the active customer agent and day-ahead (DA) wholesale prices are known. Specifically, we consider a 24 h horizon for day-ahead RTP.

1.2. Literature review and contributions

Electricity retail markets and DR programs have been focused by many researchers in recent years. Some have discussed customers' participation in different demand response programs [1–11]. The demand-side response to the offered prices [8–16] and electricity procurement for large consumers [17] are other subjects of research. Ref. [16] has analyzed the price elasticity of different categories of customers. References [3,4] have presented an economic load model based on the price elasticity of demand and the effect of incentives and penalties of DR programs on the customer response. Different non-linear benefit/demand functions have been proposed in [7–9]. Schweppe and his co-workers have developed the concept of spot pricing of electricity to evaluate the variable costs of electric energy on an hourly basis and proposed three responsive load models, namely linear, potential and exponential demand functions [8]. Ref. [9] has modeled customers' response to the optimal real time prices for the electricity utilizing different mathematical load models. Adjusting the hourly load level of a given consumer in response to hourly electricity prices is modeled in ref. [10]. The latter study applies up/down ramping rates to model changes in customer load. Ref. [14] has presented an acceptance function based on the acceptable energy costs for different clusters of customers. Also, ref. [15] has modeled the customer's behavior against the offered fixed prices for monthly bilateral contracts using a type of market share function. In the previous works, the load curve has been divided into separate periods such as peak, off-peak and valley hours and predetermined price elasticities are considered for the purpose of evaluating hourly changes in customer's demand. To the best of our knowledge, despite of applying the concept of price elasticity of demand in technical literature, there is no sign of adopting demand models in order to extract the price elasticities from demand functions based on the main definition of elasticity.

A wide range of agent theory applications have been reported in power engineering studies from long term planning to real time operation [18–22]. Ref. [20] has presented a general model of the interaction among competitor retailers and heterogeneous consumers on a MAS basis using QL approach to model the behavior of the players.

This paper presents an innovative approach in modeling customer's behavior in response to different DR programs and employs the developed CDR model in day-ahead real time pricing in an agent-based retail environment using an optimization technique which is based on the principles of QL method in a gradual trial-and-error process composed of pricing and experiencing customers' response. The main contributions of the paper are outlined in the following:

- A continuous, differentiable composite demand model composed of weighted demand functions is introduced in which the weights of demand models are determined according to the results of regression-based demand curve fittings. The CDF can be employed to represent the demand model of almost all consuming sectors due to the possibility of adjusting the weighting coefficients using the historical data of implementing DR programs for the target population of customers.
- Dynamic self elasticities are introduced which are extracted by differentiating demand functions based on the main definition of price elasticity. These hourly elasticities are employed in modeling customer response instead of predetermined values of elasticity applied in previous works.
- A comprehensive demand response model based on the proposed CDF and dynamic price elasticities is developed which represents the hourly changes in the customer's demand according to his/her load level, the customers' demand model and dynamic price elasticities of demand as well as the offered prices, incentives and penalties in different DR programs.

1.3. Paper structure

The remaining parts of the paper are structured as follows. Section 2 presents modeling of customer response to DR programs based on the proposed CDR model. Section 3 is on day-ahead real time pricing for active customer agent. Section 4 is assigned to the numerical studies. Finally, Section 5 concludes the paper.

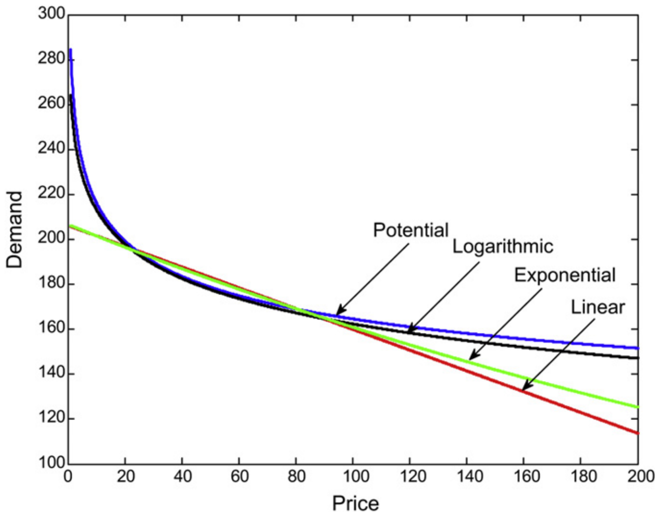

Fig. 1. Different mathematical functions of demand vs. price.

2. Modeling customer response to DR programs

This section is devoted to the modeling of customers' participation in different DR programs. Customers in smart power grids, adjust their energy consumption based on the following main factors:

- How does the customer use the electricity and how much benefit does it acquire for him/her?
- How much does the electricity cost?
- Which demand response programs can he/she participate in? How much incentive does the customer receive for the load reduction and how much does he/she have to pay for not committing to the obligations addressed in the adopted DR program?

DR programs are categorized in two groups of time-based and incentive-based programs [4]. Time-based programs encourage customers to adjust their load profiles according to the varying prices of the electricity while the incentive-based ones provide motivative payments for the customers who reduce electricity use at certain hours. Participation of customers in different DR programs is mathematically modeled based on the customer's obtained benefit for the use of electricity as well as the offered incentives and the specified penalties.

2.1. Demand and benefit functions

Different representations of demand vs. price have been addressed in previous works [7–9,23]. Fig. 1 shows linear, potential, logarithmic and exponential representations of demand vs. price function. Linear demand function ($i.e. d^{lin}(h) = a_{lin} + b_{lin}p(h)$) is the simplest and one of the most widely used models of the responsive load [9] which illustrates the customer's demand as a linear function of price. Customer's benefit from the use of electricity corresponding to linear demand function has been addressed in ref. [3,4,7–9] as follows:

$$B^C\left(d^{lin}(h)\right) = B_0^{lin}(h) + p_0(h)\left[d^{lin}(h) - d_0^{lin}(h)\right] \times \left\{1 + \frac{d^{lin}(h) - d_0^{lin}(h)}{2E(h)d_0^{lin}(h)}\right\} \tag{1}$$

Potential demand function is a widely used model with different versions of representation ($e.g.$ $d^{ptn}(h) = a_{ptn}(p(h))^{b_{ptn}}$). Ref. [7] has proposed customer benefit function corresponding to potential demand function as the following:

$$B^C\left(d^{ptn}(h)\right) = B_0^{ptn}(h) + \frac{p_0(h)d^{ptn}(h)}{1 + E^{-1}(h)}\left(\frac{d^{ptn}(h)}{d_0^{ptn}(h)}\right)^{E^{-1}(h)} \tag{2}$$

Logarithmic demand function ($i.e.$ $d^{log}(h) = a_{log} + b_{log}\ln(p(h))$) corresponds to benefit equation as represented by Eq. (3) [8].

$$B^C\left(d^{log}(h)\right) = B_0^{log}(h) + p_0(h)d_0^{log}(h)E(h) \times \left\{exp\left(\frac{d^{log}(h) - d_0^{log}(h)}{E(h)d_0^{log}(h)}\right) - 1\right\} \tag{3}$$

The exponential demand function ($i.e.$ $d^{exp}(h) = a_{exp}exp(b_{exp}p(h))$) has been proposed by Schweppe et al. [7,8]. The corresponding benefit function has been illustrated as:

$$B^C\left(d^{exp}(h)\right) = B_0^{exp}(h) + p_0(h)d^{exp}(h) \times \left\{1 + \frac{1}{E(h)}\left[\ln\left(\frac{d^{exp}(h)}{d_0^{exp}(h)}\right) - 1\right]\right\} \tag{4}$$

2.2. Economic responsive load model according to the offered incentives and penalties

An economic responsive load model has been proposed in [3,4] which represents the impacts of participating in a time-based and/or an incentive-based DR program on the customer's load profile. Here, the model represented in [3,4] is used as the basis responsive load model to structure the comprehensive demand response model. If the customer's obtained benefit for consuming the hourly demand of $d(h)$ be represented by $B^C(d(h))$, then the customer's benefit through energy consumption together with participating in DR program ($S(d(h))$) will be as follows [3,4]:

$$S(d(h)) = B^C(d(h)) - d(h) \times p(h) + inc(h) \times (d_0(h) - d(h)) - pen(h) \times [CL(h) - (d_0(h) - d(h))] \tag{5}$$

where $CL(h)$ denotes the obligated level of hourly load reduction in accordance with the DR contract. The customer's maximum benefit would be determined by $\frac{\partial S(h)}{\partial d(h)} = 0$ which results in:

$$\frac{\partial B^C(d(h))}{\partial d(h)} = inc(h) + p(h) + pen(h) \tag{6}$$

Eq. (6) addresses differentiating the benefit function for the purpose of modeling customer response. Adopting the linear model of customers' demand, the benefit function in the above equation has been substituted by Eq. (1) in [3,4] which has resulted in Eq. (7).

$$p_0(h)\left[1 + \frac{d^{lin}(h) - d_0^{lin}(h)}{E(h)d_0^{lin}(h)}\right] = inc(h) + p(h) + pen(h) \tag{7}$$

Consequently,

$$d^{lin}(h) = d_0^{lin}(h)\left[1 + E(h)\frac{p(h) - p_0(h) + inc(h) + pen(h)}{p_0(h)}\right] \tag{8}$$

Eq. (8) represents the hourly changes in customer demand according to the adopted time-based and/or incentive-based DR program in which he/she participate. Avoiding any conflict between the demand vs. price functions ($i.e.\ d^{\mathcal{F}}(h) = \mathcal{F}(p(h)), \forall \mathcal{F} \in \{lin, ptn, exp, log\}$) and the customers' demand functions in response to different DR programs, hereinafter the demand response functions are referred to as $DR^{\mathcal{F}}(h)$.

However, the economic responsive load model can be employed in simulating the behaviors of customers whose electricity demand may be represented by each of mathematical functions (i.e. linear, potential, logarithmic, exponential and other functions whichever would be proposed in future demand studies). Eq. (9) illustrates the results of differentiating the benefit functions in accordance with the above mentioned demand models.

$$\frac{\partial B^C\left(d^{\mathcal{F}}(h)\right)}{\partial d^{\mathcal{F}}(h)} = \begin{cases} p_0(h)\left[1 + \dfrac{d^{lin}(h) - d_0^{lin}(h)}{E(h)d_0^{lin}(h)}\right] & \text{if } d^{\mathcal{F}}(h) = d^{lin}(h) \\ p_0(h)\left(\dfrac{d^{ptn}(h)}{d_0^{ptn}(h)}\right)^{E^{-1}(h)} & \text{if } d^{\mathcal{F}}(h) = d^{ptn}(h) \\ p_0(h)exp\left(\dfrac{d^{log}(h) - d_0^{log}(h)}{E(h)d_0^{log}(h)}\right) & \text{if } d^{\mathcal{F}}(h) = d^{log}(h) \\ p_0(h)\left[1 + \dfrac{1}{E(h)}\, ln\left(\dfrac{d^{exp}(h)}{d_0^{exp}(h)}\right)\right] & \text{if } d^{\mathcal{F}}(h) = d^{exp}(h) \end{cases} \tag{9}$$

Assuming particular $E^{\mathcal{F}}(h)$ for $B^C(d^{\mathcal{F}}(h))$, substituting $\frac{\partial B^C(d^{\mathcal{F}}(h))}{\partial d^{\mathcal{F}}(h)}$ in Eq. (6) and solving the equations for $d^{\mathcal{F}}(h)$ result in the corresponding demand response models as follows:

$$DR^{\mathcal{F}}(h) = \begin{cases} d_0^{lin}(h)\left[1 + E^{lin}(h)\dfrac{p(h) - p_0(h) + inc(h) + pen(h)}{p_0(h)}\right] & \text{if } DR^{\mathcal{F}}(h) = DR^{lin}(h) \\ d_0^{ptn}(h)\left(\dfrac{p(h) + inc(h) + pen(h)}{p_0(h)}\right)^{E^{ptn}(h)} & \text{if } DR^{\mathcal{F}}(h) = DR^{ptn}(h) \\ d_0^{log}(h)\left[1 + E^{log}(h)\ln\left(\dfrac{p(h) + inc(h) + pen(h)}{p_0(h)}\right)\right] & \text{if } DR^{\mathcal{F}}(h) = DR^{log}(h) \\ d_0^{exp}(h)exp\left(E^{exp}(h)\dfrac{p(h) - p_0(h) + inc(h) + pen(h)}{p_0(h)}\right) & \text{if } DR^{\mathcal{F}}(h) = DR^{exp}(h) \end{cases} \tag{10}$$

Eq. (10) demonstrates the behaviors of customers whose electricity demand is represented by $d^{\mathcal{F}}(h)$ in response to the time-based and/or incentive-based DR program in which they participate. Time-based DR programs encourage the participants to modify their electricity consumption by means of varying energy prices. Therefore, there are no incentives/penalties in these programs. In order to model customer response to time-based DR programs, it is necessary to set $inc(h) = 0,\quad pen(h) = 0$ in the above equation. Similarly, simulating customer response to incentive-based DR programs makes it essential to set $p(h) = p_0(h)$in $DR^{\mathcal{F}}(h)$.

2.3. Dynamic price elasticities

Ref. [3,4,9] have employed the price elasticity of demand as fixed values and studied demand response models substituting those assumed values as $E(h)$ in demand functions. These price elasticities are independent from demand functions. However as represented in Eq. (11), the main definition of price elasticity implies differentiating the demand vs. price function. Therefore, if a customer's demand is modeled by a mathematical function, its price elasticity will be also defined based on the adopted model referring to Eq. (11) as the basic definition.

$$E(h) = \frac{p_0(h)\partial d(h)}{d_0(h)\partial p(h)} \tag{11}$$

Differentiating a demand function leads to the hourly price elasticities according to the adopted demand model. Here, the hourly elasticity is referred to as dynamic price elasticity and applied in its corresponding benefit function. Setting $E^{\mathcal{F}}(h) = \frac{p_0(h)\partial d^{\mathcal{F}}(h)}{d_0^{\mathcal{F}}(h)\partial p(h)}$ for each demand model and solving it for $d^{\mathcal{F}}(h) = \mathcal{F}(p(h)), \forall \mathcal{F} \in \{lin, ptn, log, exp\}$ yields the corresponding dynamic price elasticity. Eq. (12) represents the dynamic price elasticities of different demand model.

$$E^{lin}(h) = b_{lin}\frac{p_0(h)}{d_0^{lin}(h)} = b_{lin}\frac{p_0(h)}{a_{lin} + b_{lin}p_0(h)} \tag{12a}$$

$$E^{ptn}(h) = \left[a_{ptn}b_{ptn}(p_0(h))^{b_{ptn}-1}\right]\frac{p_0(h)}{d_0^{ptn}(h)} = b_{ptn} \tag{12b}$$

$$E^{log}(h) = \left[\frac{b_{log}}{p_0(h)}\right]\frac{p_0(h)}{d_0^{log}(h)} = \frac{b_{log}}{a_{log} + b_{log}\ln\left(p_0(h)\right)} \tag{12c}$$

$$E^{ptn}(h) = \left[a_{exp}b_{exp}exp\left(b_{exp}p_0(h)\right)\right]\frac{p_0(h)}{d_0^{exp}(h)} = b_{exp}p_0(h) \tag{12d}$$

2.4. Comprehensive demand response model

In different studies addressed in the literature review, various types of customer demand functions have been proposed and applied in representing customer response. The load profiles of each customer reflect his/her energy use style. Although the mentioned demand functions exhibit similar trends for intermediate prices, however divergent customer demands are demonstrated by them for prices out of the intermediate range. Even unrealistic customer benefits for very small values of demand are evaluated by non-linear benefit functions. Also, linear demand function indicates a price cap above which the customers curtail their entire electricity load [9]. Despite these shortcomings, differences in the customers' energy consuming habitudes and purposes can be modeled by different demand/benefit functions. For example, potential demand function may represent the behavior of customers who consume energy even at high prices due to their loads of high importance. Also, linear function models the responses of customers with interruptible loads. Therefore, as it is shown in Eq. (13), a composite demand model based on a weighted combination of different demand functions is proposed so as to better represent the demand function of a consuming group with diverse load profiles.

$$d(h)=w_{lin}d^{lin}(h)+w_{ptn}d^{ptn}(h)+w_{log}d^{log}(h)+w_{exp}d^{exp}(h) \tag{13}$$

where, w_F denotes the assigned weight to the respected demand function according to the studies on the historical load profiles in the consuming sector. Determining the weighting coefficients in the proposed CDF is addressed in Section 2.5. As illustrated by Eq. (14), substituting $d^{\mathcal{F}}=\mathcal{F}(p(h))$ in Eq. (13) yields the proposed CDF as a function of hourly prices.

$$\begin{aligned} d(h) = {} & w_{lin}[a_{lin}p(h)+b_{lin}]+w_{ptn}a_{ptn}(p(h))^{b_{ptn}}+w_{log}\Big[a_{log} \\ & +b_{log}ln\,(p(h))\Big]+w_{exp}a_{exp}exp\,(b_{exp}p(h)) \end{aligned} \tag{14}$$

As mentioned before, expanding the responsive load model proposed in [3,4] results in Eq. (10) as the mathematical representations of customer participation in DR programs according to the adopted demand models. Corresponding to the definition of composite demand model, the behavior of a DR program participant society whose demand function is modeled using Eq. (14), can be represented by a weighted combination of customer response models as:

$$\begin{aligned} DR(h)= {} & w_{lin}d_0^{lin}(h)\left[1+E^{lin}(h)\frac{p(h)-p_0(h)+inc(h)+pen(h)}{p_0(h)}\right] \\ & +w_{ptn}d_0^{ptn}(h)\left(\frac{p(h)+inc(h)+pen(h)}{p_0(h)}\right)^{E^{ptn}(h)} \\ & +w_{log}d_0^{log}(h)\left[1+E^{log}(h)\left\{ln\left(\frac{p(h)+inc(h)+pen(h)}{p_0(h)}\right)\right\}\right] \\ & +w_{exp}d_0^{exp}(h)exp\left(E^{exp}(h)\frac{p(h)-p_0(h)+inc(h)+pen(h)}{p_0(h)}\right) \end{aligned} \tag{15}$$

According to the concept of dynamic price elasticities, Eq. (12) should be substituted in Eq. (15). Therefore, the customers' response to the adopted DR program which appears in their modified demand will be as:

$$\begin{aligned} DR(h) = {} & w_{lin}d_0^{lin}(h)\left[1+b_{lin}\frac{p(h)-p_0(h)+inc(h)+pen(h)}{a_{lin}+b_{lin}p_0(h)}\right]+w_{ptn}d_0^{ptn}(h)\left(\frac{p(h)+inc(h)+pen(h)}{p_0(h)}\right)^{b_{ptn}} \\ & +w_{log}d_0^{log}(h)\left[1+\frac{b_{log}}{a_{log}+b_{log}ln\,(p_0(h))}\left\{ln\,\left(\frac{p(h)+inc(h)+pen(h)}{p_0(h)}\right)\right\}\right] \\ & +w_{exp}d_0^{exp}(h)exp\,(b_{exp}[p(h)-p_0(h)+inc(h)+pen(h)]) \end{aligned} \tag{16}$$

Eq. (16) represents the comprehensive demand response model which is applicable in evaluating customer response to different types of DR programs according to the customer's load level, his/her demand model and dynamic price elasticities of demand as well as the offered prices, incentives and penalties corresponding to the DR contract. The proposed process of providing composite demand function and the CDR model is illustrated in Fig. 2.

2.5. Weighting coefficients

Expanded DR implementation is a result of coordinated actions along the electricity supply chain including regulators, system operators and load-serving entities (local distribution companies and REPs). These entities cooperate in implementing DR programs from regulating supportive market rules to supplying communication technologies and encouraging customers to participate in DR programs [2]. Suppose that one of these DR providers aims at evaluating the response of a consuming sector to each of DR programs. One of the suitable tools widely used in power system studies especially for the purpose of modeling load profiles is the regression-based curve fitting technique [24]. Fitting process requires a parametric model that relates the response data to the predictor data with one or more coefficients. In the regression approach, the relationship between available historical data and demand response models are formulated as linear/non-linear equations.

In order to model the customer response to DR programs, it is necessary to determine $a_{\mathcal{F}}, b_{\mathcal{F}}$ coefficients of the demand functions; $d^{\mathcal{F}}(h) = \mathcal{F}(p(h))$ at the first stage, and then associate proper weighting coefficients ($w_{\mathcal{F}}$) at the second stage. The requisite coefficients are extractible through fitting $DR^{\mathcal{F}}(h)$ to the historical data of implementing the DR program as follows:

$$y_k = DR^{\mathcal{F}}(h) + \epsilon \tag{17}$$

where y_h denotes the kth sample of the historical data related to the implementation of the concerned DR program. DR providers have to conduct the process of fitting $DR^{\mathcal{F}}(h), \forall \mathcal{F}\emptyset \in \{lin, ptn, log, exp\}$ to the historical customer response curves separately for each type of demand function. Here, the coefficients are estimated by the least squares method (LSM) through fitting each of $DR^{\mathcal{F}}(h)$ functions for the whole historical data in four independent regression procedures associated to linear, potential, logarithmic and exponential demand response models. The next stage is to fit the proposed CDR model to the historical data so as to determine the best weighting coefficients ($w_{\mathcal{F}}$) through LSM-based solution of Eq. (18).

$$y_k = \sum_{\mathcal{F}\in\{lin,ptn,log,exp\}} w_{\mathcal{F}}DR^{\mathcal{F}}(h) + \epsilon \tag{18}$$

As it is shown in Eq. (18) the customer response is modeled as a linear combination of (not necessarily linear) functions of the predictors, plus a random error ϵ. Fig. 3 shows the regression-based process of extracting demand response models from the historical data of DR implementation.

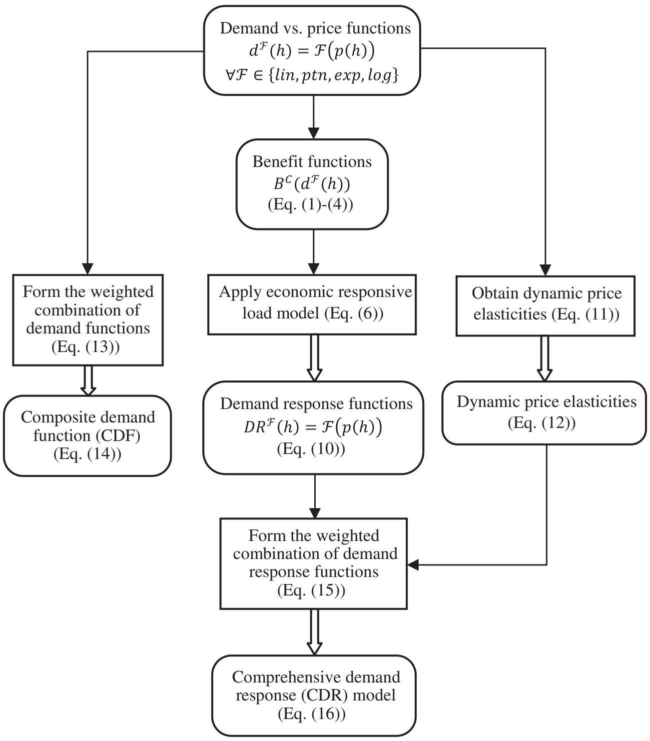

Fig. 2. Extracting CDF and CDR model from individual demand functions.

Based on the benefit functions (Eqs. (1)–(4)) and the economic responsive load model (Eq. (6)), customer response to all types of DR programs was represented by Eq. (10). Furthermore, the behavior of a DR program participant group was simulated by the CDR model. The first stage provides individual demand response functions (i.e. $DR^{\mathcal{F}}(h)$ for DR provider while the second stage results in a comprehensive DR model (i.e. $DR(h)$) for the purpose of representing the behavior of a group of customers facing the hourly

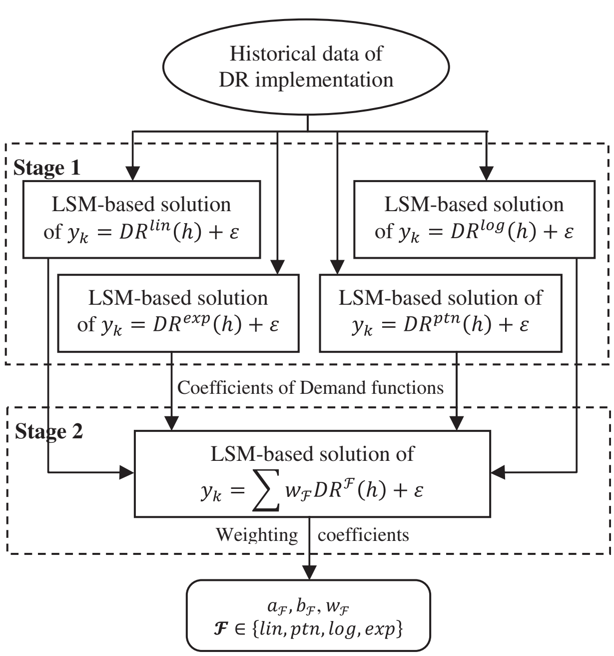

Fig. 3. The regression-based process of extracting coefficients.

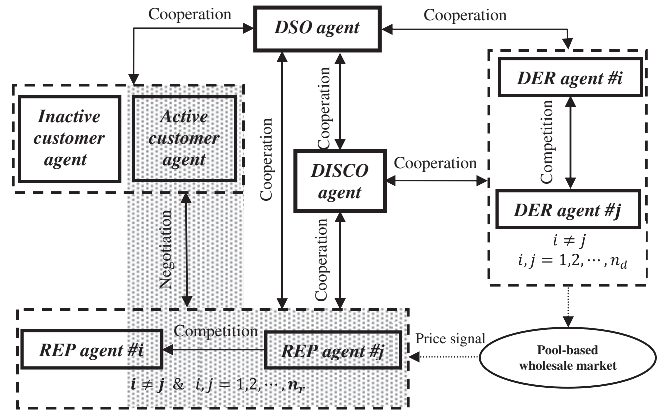

Fig. 4. Multi-agent based electricity retail market structure.

changes in the electricity price and/or the offered incentive payments for load reduction. Determining $a_{\mathcal{F}}, b_{\mathcal{F}}$ coefficients of the demand functions and weighting coefficients ($w_{\mathcal{F}}$) in separate stages, makes it possible to compare the performances of individual demand response functions and the weighted combination of them, in predicting customer response to the adopted DR program. The composite demand function and the CDR model utilize the associated weights in order to better represent the behavior of participant customers with divergent load profiles and energy consumption habitudes. The demand-side reaction to DR programs in almost all consuming sectors can be represented by the proposed CDR model due to the possibility of adjusting the weighting coefficients using the regression-based historical data fitting approaches.

3. Day-ahead real time pricing based on CDR model of customer response

As mentioned before, in smart grids active customers receive real time pricing information and adjust their effective demands accordingly. As shown in Eq. (16), the CDR model represents the hourly changes in customer's demand according to the DR program in which he/she enroll. This section is devoted to retailing inter-relation between active customer agent and its relevant REP agent in an agent-based retail market environment.

3.1. Agent-based retail environment

A complete structure of an agent-based electricity retail market is depicted in Fig. 4 including five defined types of heterogeneous communicating agents. We consider a pool-based wholesale market in which Distributed Generators (DGs) offer their generation as well as large scale power plants. REP agents are economic enterprises which enroll electricity customers, offer various demand response programs and encourage them to hold next contracts so as to gain as much profit as possible through marketing activities. Assigning an agent to each customer is the most exact representation of the consuming side in the retail market. However, due to dimensionality problem, in the proposed agent-based environment it is considered that for each cluster of customers an agent be assigned. From REP point of view, they are categorized according to their adopted pricing schemes. As it can be seen in Fig. 4, there are two customer agents corresponding to pricing patterns with predetermined rates like fixed and TOU prices (inactive customer agent) and real time pricing pattern (active customer agent). Customers of active customer agent usually purchase a portion of their demand through bilateral contracts and then bid in day-ahead and spot markets to procure their surplus electric power need. The active customers monitor other pricing alternatives and different provided services and usually participate in a variety of demand response programs.

However as it is shown in the shaded part of Fig. 4, the focus of this paper is on the bidirectional RTP interrelation between active customer agent and one of REP agents which have an indirect competition interaction with other REPs via customer response to the offered prices. REP agent's intelligence appears in the ability of learning the optimal pricing policy by experiencing its impact on retailing profit which is modeled using QL approach.

3.2. RTP model based on customers' response to the offered real time prices

In this paper, it is considered that the REP agent purchases energy in DA market, adopts real time pricing and offers hourly DA prices to its active customers based on energy procurement cost, the behavior of competitors, the adopted retail strategy, its expected benefit, etc. RTP is a time-based DR program in which participant customers adjust their load profiles according to real time prices. Therefore, there is no incentive payment offered by the DR provider or penalty received by it. The REP agent proposes DA prices and experiences subsequent customers' reactions by observing active agent's demand. RTP model is formulated as:

$$\text{Maximize}_{p_i(h)} B^{REP}(p_i(h)) = DR_i(h) \times (p_i(h) - p_w(h))$$
$$h = 1,2,\ldots,24 \ \& \ i = 1,2,\cdots,L \tag{19a}$$

subject to:

$$\begin{aligned} DR_i(h) = {} & w_{lin} d_0^{lin}(h)\left[1 + b_{lin}\frac{p_i(h)-p_0(h)}{a_{lin}+b_{lin}p_0(h)}\right] + w_{ptn} d_0^{ptn}(h)\left(\frac{p_i(h)}{p_0(h)}\right)^{b_{ptn}} \\ & + w_{log} d_0^{log}(h)\left[1 + \frac{b_{log}}{a_{log}+b_{log}\ln(p_0(h))}\left\{\ln\left(\frac{p_i(h)}{p_0(h)}\right)\right\}\right] \\ & + w_{exp} d_0^{exp}(h)\exp\left(b_{exp}[p_i(h)-p_0(h)]\right) \end{aligned} \tag{19b}$$

$$p^{min}(h) \le p_i(h) \le p^{max}(h) \tag{19c}$$

where $i = 1,2,\cdots,L$ denotes the number of learning iteration, $p_i(h)$ is the candidate price of electricity to be offered to the active customer agent in the ith iteration of learning process and $DR_i(h)$ represents the modeled customer response to the offered price. Hourly price of the electricity in DA wholesale market is denoted by $p_w(h)$ which impresses energy procurement cost for the REP agent. This model is defined for each of the 24 h of the next day. Note that in each hour, initial demands are known and DA wholesale price is forecasted while RTP rate is the variable to be determined. Eq. (19b) demonstrates the active customer agent's response to the offered real time prices ($p_i(h)$) according to its initial load level and initial retail price of electricity. Constraint (19c) establishes minimum and maximum limitations for RTP rates according to the predetermined lower and upper limits for the hourly retail prices.

3.3. QL-based day-ahead real time pricing

The REP agent acts as a learner agent not aware of its environment's mathematical model. It does not know which price to select in order to maximize its benefit. Therefore, it tries to discover which options yield the most subsequent rewards or penalties, gradually. The REP agent can learn from its past experienced strategies which can be computationally implemented by using a Q-Learning algorithm. QL is learning how to map situations to actions so as to maximize a numerical reward signal [25]. In this study, one-step QL is applied as REP agent's learning approach so as to reach optimized benefit while satisfying active customers.

Let $S = \{s_1, s_2, \ldots, s_{n_s}\}$ be the finite set of possible states in RTP process and $A = \{a_1, a_2, \ldots, a_{n_a}\}$ be the finite set of admissible actions the agent can take where n_s and n_a represent the number of states and actions in the learning process, respectively. At each time step t_i, the agent senses the current state $s_i \in S$ and on that basis selects an action $a_i = a \in A$. In each state, three possible actions are defined as represented by Eq. (20).

$$A = \{a_+, a_-, a_0\} \tag{20a}$$

where,

$$p_i(h) \xrightarrow{a_+} (p_{i+1}(h) = p_i(h) + \Delta p) \tag{20b}$$

$$p_i(h) \xrightarrow{a_-} (p_{i+1}(h) = p_i(h) - \Delta p) \tag{20c}$$

$$p_i(h) \xrightarrow{a_0} (p_{i+1}(h) = p_i(h)) \tag{20d}$$

Here, a combination of soft-max and greedy policies is applied in pricing procedure. In each stage, based on the adopted policy, an action is selected. The above mentioned policies are based on Boltzmann distribution (Eq. (21)) and maximum probabilities, respectively.

$$p(s_i, a_i, i) = \frac{e^{Q_{i-1}(s_i,a_i)/T_i}}{\sum_{j=1}^{n_a} e^{Q_{i-1}(s_i,a_i)/T_i}} \tag{21}$$

The temperature T_i usually decreases during the learning iterations. Here, the reduction pattern is as the following equation:

$$\begin{aligned} T_i &= T_1(1-(i-1)/T_1) + 1e-5 \\ T_1 &= L \end{aligned} \tag{22}$$

The updated price as a result of the adopted action affects the retailing beneit value and accordingly leads the agent to a new state of pricing strategy learning. In this study, three states are conceivable for the agent, gaining more benefit, less benefit or no change as represented by Eq. (23).

$$S = \{s_+, s_-, s_0\} \tag{23a}$$

where,

$$s_i = \begin{cases} s_+ = +1 & \text{if } B^{REP}(p_i(h)) > B^{REP}(p_{i-1}(h)) \\ s_0 = 0 & \text{if } B^{REP}(p_i(h)) = B^{REP}(p_{i-1}(h)) \\ s_- = -1 & \text{if } B^{REP}(p_i(h)) < B^{REP}(p_{i-1}(h)) \end{cases} \tag{23b}$$

REP agent receives an immediate reward (r_i) which is proportional to the resulted change in its benefit value. Accordingly, the current state of the agent updates to the new state (s_{i+1}). Eq. (24)

represents the reward assigned to the action a_i from the old state s_i which has caused changes in REP agent's obtained benefit.

$$r_i = 100 \times s_{i+1} + 1e - 3 \times (s_{i+1} + 1) \tag{24}$$

The offered reward impresses action-state value function $Q(s_i,a_i)$ as represented by Eq. (25).

$$Q(s_i, a_i) = Q(s_i, a_i) + a[r_i + \gamma \max_a Q(s_{i+1}, a) - Q(s_i, a_i)] \tag{25}$$

These functions determine the most probable actions for the next play and are applied in taking the next action based on the mentioned policies. A limited number of stages (L) are allowed for the learning procedure as its termination criterion. The offered price and the related benefit reach to their final values as the learning process terminates. The flowchart of the proposed QL-based method of real time pricing for the active customer agent is presented in Fig. 5.

The above pricing process is repeated for the whole day on an hourly basis in order to determine optimum DA real time prices.

4. Numerical studies

In this study, the time-based RTP program is investigated while the required data is extracted from the day-head market of New England, Connecticut [26]. For the similar hours in two consequent days, it is assumed that the impacts of parameters affecting the load are ignorable except the price change. Therefore, the historical data of hourly price and demand in two consequent days may reflect the impact of varying prices on the modified electricity use of RTP program participants. Accordingly, the demand and price data at hth hour of the $(i-1)$th day can be used as initial demand and price values for the same hour of ith day. Modeling of customer participation in RTP program are conducted for two types of historical demand curves which represent samples from

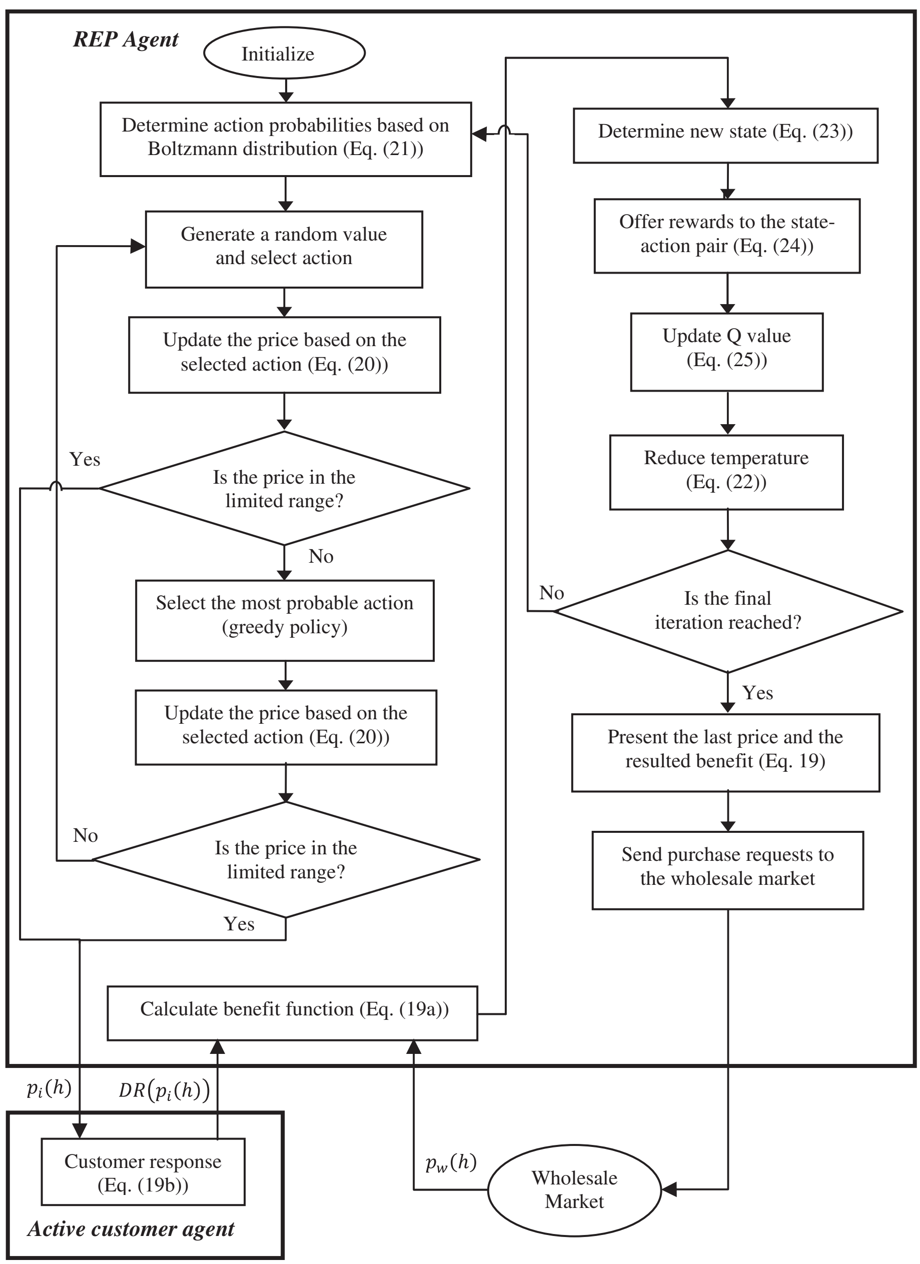

Fig. 5. The flowchart of the proposed QL-based day-ahead real time pricing.

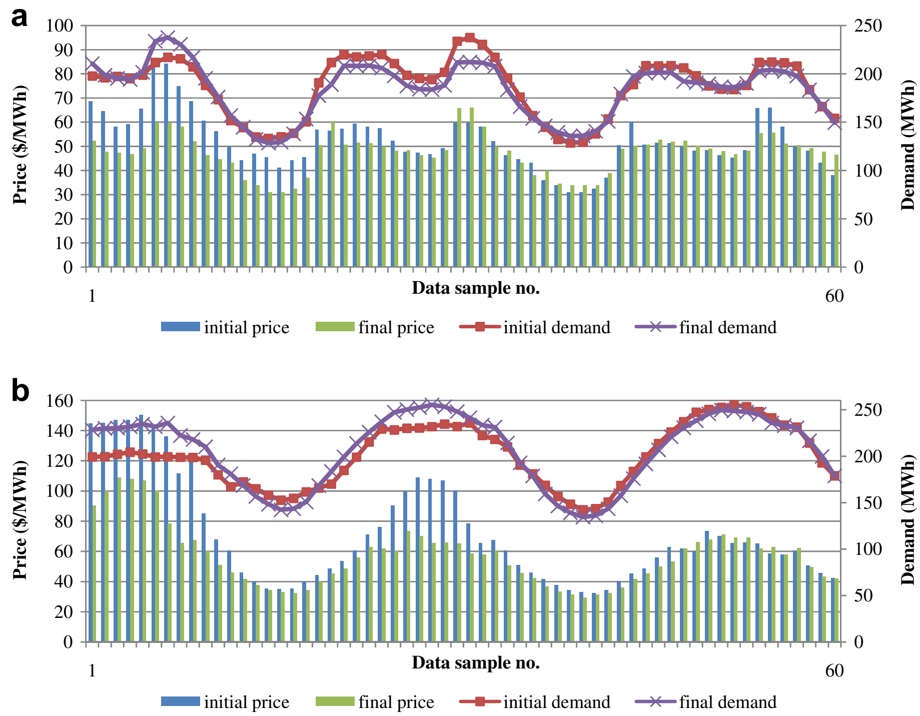

Fig. 6. Market data, a) winter season (16–19 Feb. 2010), b) summer season (16–19 Aug. 2010).

the summer and winter load profiles. The historical load data correspond to the hourly demand and price data for 16–19 Feb. 2010 as the sample data for the winter season, and 16–19 Aug. 2010 as the sample data for the summer season. Demand response functions (Eq. (10)) and the proposed CDR model are separately fitted to the historical data and day-ahead real time pricing is conducted for two target days of 20 Feb. 2010 and 20 Aug 2010. The performances of individual demand response functions (i.e. $DR^{\mathcal{F}}(h)$) and the weighted combination of them (i.e. $DR(h)$) in predicting customer response to RTP program are compared based on the demand and price data related to the target days. Historical market data which are used in fitting process is provided in Fig. 6. In RTP process, the minimum and maximum retail prices are considered as $p^{min}(h) = p_w(h)$ and $p^{max}(h) = 1.5 \times p_w(h)$. Also, the assumed parameters in the equations are as the following: $\alpha = 0.2$, $\gamma = 0.95$, $L = 1000$.

4.1. Demand curve fitting and dynamic price elasticities

In order to evaluate the performance of the proposed CDR model, it is necessary to find the best fitted mathematical functions to the historical demand and price data at the first stage and then, determine proper weighting coefficients so as to compare the results of predicting customer response based on the structured $DR^{\mathcal{F}}(h), \forall \mathcal{F} \in \{lin, ptn, log, exp\}$ and the proposed $DR(h)$. Fitting demand response function to the historical market data results in estimates of the model coefficients as presented in Table 1.

As it is shown in Fig. 7, dynamic price elasticities of demand are dependant to demand vs. price functions whose coefficients vary according to the historical load profiles.

As it is shown in Fig. 7, linear demand function exhibits greatest absolute values of price elasticity for high rates of electricity price while the price elasticity of potential demand function is a fixed value independent from the hourly price of electricity and logarithmic demand function represents the electricity demand with intermediate price elasticities. Furthermore, the summer load shows greater decreases (i.e. higher absolute elasticities) in comparison with the winter load in the same electricity prices. It demonstrates the importance of the electrical devices which are applied in the winter season. This is due to the wide usage of the heating equipment in the cold region of Connecticut in winter days.

4.2. Weighting coefficients and CDR models

Weighted combination of demand functions makes it possible to represent the response of a group of customers composed of heterogeneous customers with different load profiles. Using data samples from the winter and summer historical load data, LSM results in the weighting coefficients presented in Table 2 as the

Table 1
Fitting demand functions to the historical data.

Seasons	Winter season			Summer season		
Coefficients	a_F	b_F	$d^F = F(P)$	a_F	b_F	$d^F = F(P)$
Linear	209.381	−0.308	$d^{lin}(h) = 209.381 - 0.308P(h)$	209.429	−0.441	$d^{lin}(h) = 209.429 - 0.441P(h)$
Potential	294.243	−0.057	$d^{ptn}(h) = 294.243(P(h))^{-0.057}$	209.005	−0.215	$d^{ptn}(h) = 209.005\ (p(h))^{-0.215}$
Logarithmic	272.045	−13.397	$d^{log}(h) = 272.045 - 13.397\ \ln P(h)$	208.777	−23.566	$d^{log}(h) = 208.777 - 23.566\ \ln P(h)$
Exponential	210.694	−0.001	$d^{exp}(h) = 210.694 - 0.001e^{\ p(h)}$	209.565	−0.003	$d^{exp}(h) = 209.565 - 0.003e^{\ P(h)}$

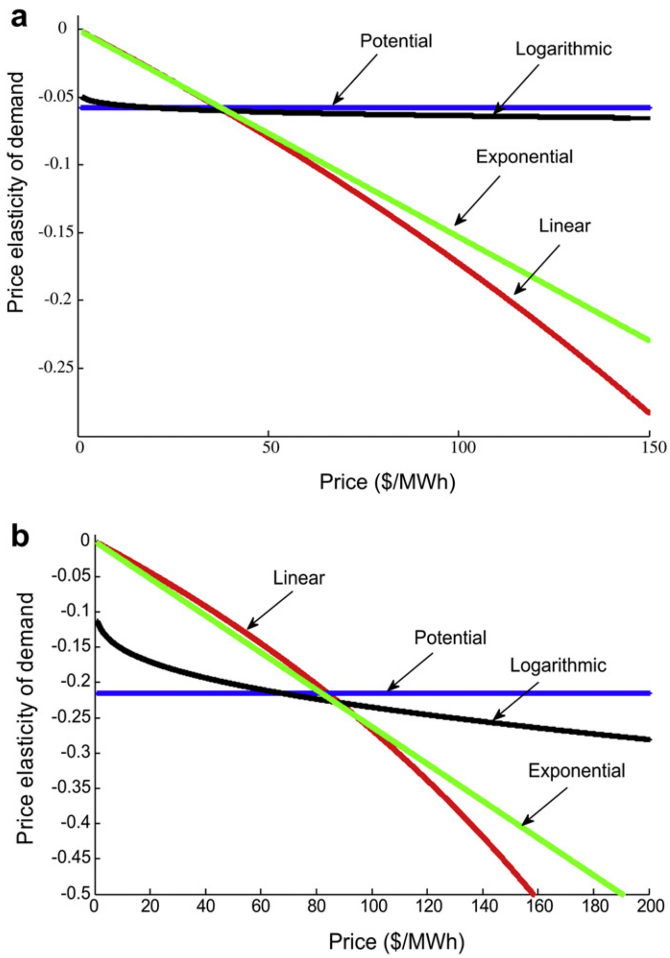

Fig. 7. Price elasticity of demand, a) winter season, b) summer season.

associated weights to the demand response terms in the CDR model (Eq. (16)). Fig. 8 shows the winter and summer CDF models as the customer response models according to the hourly retail prices.

4.3. Evaluating the proposed CDR model

As it was mentioned before, determining the requisite coefficients of the demand functions and weighting coefficients in separate stages, makes it possible to compare the performances of individual demand response functions and the weighted combination of them, in predicting customer response to the adopted DR program. At the first stage, each $DR^{\mathcal{F}}(h)$ is fitted to the historical data and is employed in predicting customer response to the RTP program which would be implemented in target days while the second stage utilizes the $DR(h)$ for the purpose of representing customers' historical behaviors and predicting their response to the RTP program in target days. Table 3 presents the resulted percentage errors of fitting $DR^{\mathcal{F}}(h), \forall \mathcal{F} \in \{lin, ptn, log, exp\}$ and

Table 2
Weighting coefficients.

DR function	Winter season	Summer season
Linear	0.496	0.481
Potential	0.0	0.0
Logarithmic	0.031	0.109
Exponential	0.436	0.400

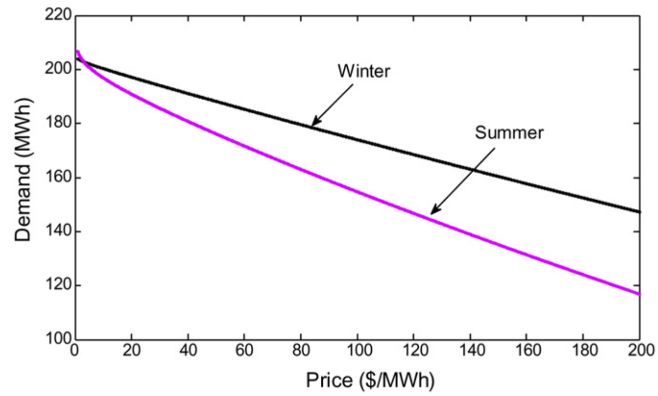

Fig. 8. Winter and summer CDR models.

$DR(h)$ to the historical data of implementing RTP program in the sample days of winter and summer seasons. Furthermore, the resulted percentage errors of predicting customer response via the above mentioned DR functions is shown in Table 3.

Investigation of the presented results in Table 3 reveals that the weighted CDR function leads to better representation of customers' historical behavior as well as improved prediction of their response to the RTP program which is implemented in two target days in comparison with the performances of the individual DR functions.

4.4. Real time pricing for active customer agent

REP agent learns to offer the best prices to its active customers through QL approach. The offered price and the related benefit converge simultaneously to the final values as the learning process reaches to its limit (i.e. *iteration no.* $= L$). The price value in the Lth iteration is considered as the optimum price determined by QL-based optimization method. However, different values of retail price are experienced by the REP agent through learning process. Fig. 9 shows the relative percent difference between the acquired price in the final iteration and the best experienced price (i.e. the price which results in the highest value of retailing benefit) in RTP for two target days in order to demonstrate the accuracy of the optimum prices obtained via QL-based optimization technique applied in this paper. As shown in Fig. 9, the relative percent difference is less than 0.6% for all hours of real time pricing in two days of winter and summer seasons. The optimum real time prices which would be offered to the active customer agent in the target days are presented in Fig. 10.

As it is shown in Fig. 10, the offered prices in the summer day are much greater than those in the winter day at certain hours. These hours correspond to the summer peak hours at which the cost to procure energy is at its highest for the REP agent. Therefore, active customer agent is charged higher prices for electricity used during peak hours of the summer day.

Table 3
Resulted errors of fitting historical data of customer response and predicting it for the target days via DR functions.

DR function	Winter season		Summer season	
	Error of fitting (%)	Error of predicting (%)	Error of fitting (%)	Error of predicting (%)
Linear	4.2443	10.0396	3.285	5.2505
Potential	4.2913	9.9451	3.9934	5.7665
Logarithmic	4.2949	9.9649	3.865	5.6748
Exponential	4.2476	10.026	3.3854	5.3129
CDR	3.7215	6.4413	3.1686	4.7409

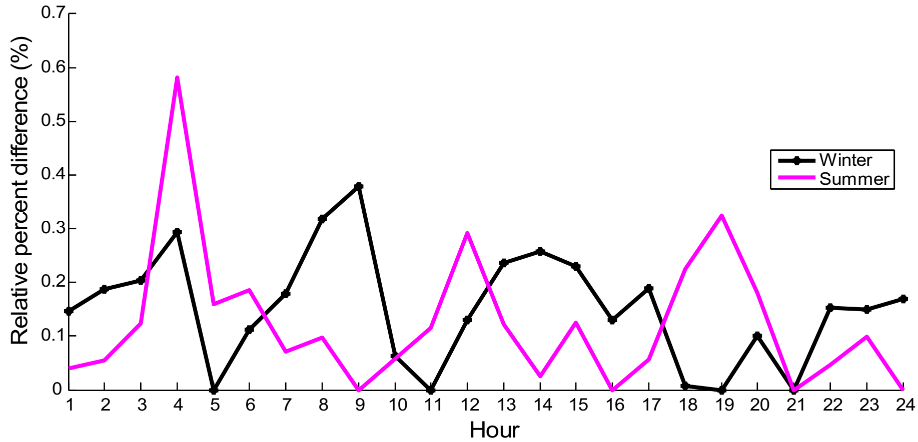

Fig. 9. Relative percent difference between the acquired price in the final iteration and the best experienced price for two target days.

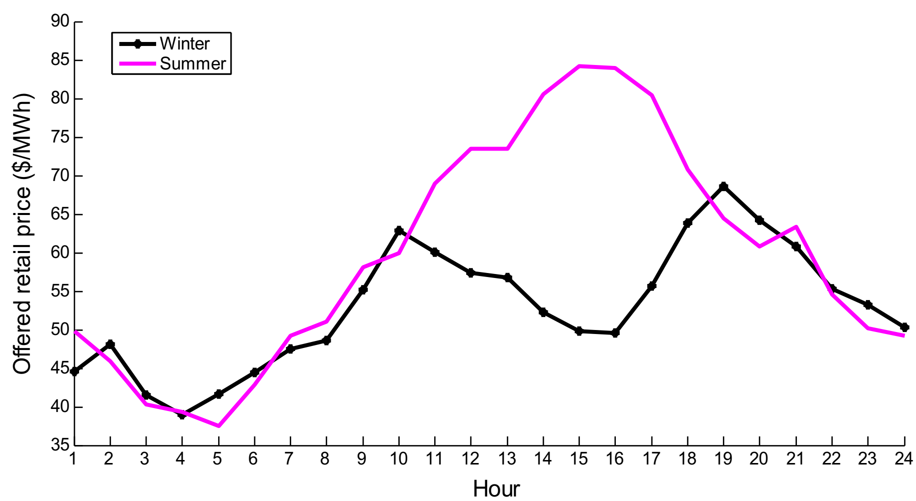

Fig. 10. Optimum hourly retail prices offered to the customers in two target days functions.

5. Conclusion

Smart power grids emphasize provision of all customers in energy market so as to utilize their potentials in improving the operation of power systems. In order to represent the behavior of a customer society with divergent energy consumption habitudes and dissimilar load profiles who participate in various DR programs, a composite demand model was proposed in this paper including some mathematical representations of demand curve such as linear, exponential, potential and logarithmic functions. The model proposed here, features with the modularity capability which enables DR providers to model different DR participant customers due to the fact that the model may be expanded using weighted combination of other mathematical functions of demand whichever would be proposed in future demand studies. The proposed CDF facilitates modeling the demand model of different customer groups due to the flexible associating appropriate weighting coefficients. Accordingly, structuring CDF resulted in a composite demand response model which was called comprehensive DR model due to the capability of modeling the hourly changes in customer demand corresponding to the customer's demand function and the price elasticity demand as well as the hourly changes in electricity prices, offered incentives and obligated penalties in different DR programs. The CDR model is structured based on the customer's obtained benefit for the use of electricity and dynamic price elasticities which were defined and extracted here by differentiating CDF components based on the main definition of self elasticity. The proposed CDR model led to better representation of customers' historical behavior against RTP program and improved prediction of their response to this type of DR program in the next days. This model was applied in simulating day-ahead real time pricing as a time-based DR program in a multi-agent retail environment. In this paper, the electricity retail market's stakeholders have been modeled by intelligent agents who make decisions and follow their predefined goals. The most beneficial dynamic prices were determined through REP agent's learning process based on principles of Q-learning method. The QL procedure was adopted in a way to incorporate different aspects of the problem such as price caps and customer response represented by the CDR model.

References

[1] Centolella P. The integration of price responsive demand into regional transmission organization (RTO) wholesale power markets and system operations. Energy 2010;35(4):1568–74.
[2] Greening LA. Demand response resources: who is responsible for implementation in a deregulated market? Energy 2010;35(4):1518–25.
[3] Aalami HA, Parsa Moghaddam M, Yousefi GR. Modeling and prioritizing demand response programs in power markets. Electric Power Systems Research 2010;80(4):426–35.

[4] Aalami HA, Parsa Moghaddam M, Yousefi GR. Demand response modeling considering interruptible/curtailable loads and capacity market programs. Applied Energy 2010;87(1):243–50.
[5] Cappers P, Goldman C, Kathan D. Demand response in U.S. electricity markets: empirical evidence. Energy 2010;35(4):1526–35.
[6] Torriti J, Hassan MG, Leach M. Demand response experience in Europe: policies, programmes and implementation. Energy 2010;35(4):1575–83.
[7] Schweppe F, Caramanis M, Tabors R. Evaluation of spot price based electricity rates. IEEE Transaction on Power Apparatus Systems PAS 1985;104(7):1644–55.
[8] Schweppe FC, Caramanis MC, Tabors RD, Bohn RE. Spot pricing of electricity. Boston MA: kluwer Ltd; 1989.
[9] Yusta JM, Khodr HM, Urdaneta AJ. Optimal pricing of default customers in electrical distribution systems: effect behavior performance of demand response models. Electric Power Systems Research 2007;77(5–6):548–58.
[10] Conejo AJ, Morales JM, Baringo L. Real-time demand response model. IEEE Transaction on Smart Grid 2010;1(3):236–42.
[11] Herterand K, Wayland S. Residential response to critical-peak pricing of electricity: California evidence. Energy 2010;35(4):1561–7.
[12] Newsham GR, Bowker BG. The effect of utility time-varying pricing and load control strategies on residential summer peak electricity use: a review. Energy Policy 2010;38(7):3289–96.
[13] Lin B, Liu J. Principles, effects and problems of differential power pricing policy for energy intensive industries in China. Energy 2011;36(1):111–8.
[14] Mahmoudi-Kohan N, Parsa Moghaddam M, Sheikh-El-Eslami MK. An annual framework for clustering-based pricing for an electricity retailer. Electric Power Systems Research 2010;80(9):1042–8.
[15] Hatami AR, Seifi H, Sheikh-El-Eslami MK. Optimal selling price and energy procurement strategies for a retailer in an electricity market. Electric Power Systems Research 2009;79(1):246–54.
[16] He YX, Yang LF, He HY, Luo T, Wang YJ. Electricity demand price elasticity in China based on computable general equilibrium model analysis. Energy 2011; 36(2):1115–23.
[17] Zare K, Parsa Moghaddam M, Sheikh-El-Eslami MK. Electricity procurement for large consumers based on Information Gap Decision Theory. Energy Policy 2010;38(1):234–42.
[18] Ma T, Nakamori Y. Modeling technological change in energy systems – from optimization to agent-based modeling. Energy 2009;34(7):873–9.
[19] Veit DJ, Weidlich A, Krafft JA. An agent-based analysis of the German electricity market with transmission capacity constraints. Energy Policy 2009; 37(10):4132–44.
[20] Bompard EF, Napoli R, Abrate G, Wan B. Multi-agent models for consumer choice and retailer strategies in the competitive electricity market. International Journal of Emerging Electric Power Systems 2007;8(2). article 4.
[21] Aquino-Lugo AA, Klump R, Overbye TJ. A control framework for the smart grid for voltage support using agent-based technologies. IEEE Transaction on Smart Grid 2011;2(1):161–8.
[22] Logenthiran T, Srinivasan D, Khambadkone AM. Multi-agent system for energy resource scheduling of integrated microgrids in a distributed system. Electric Power Systems Research 2011;81(1):138–48.
[23] Rothwell G, Gomez T. Electricity economics, regulation and deregulation. IEEE Press; May 2003. Pp 34.
[24] Song K, Baek Y, Hong DH, Jang G. Short-term load forecasting for the holidays using fuzzy linear regression method. Power Systems, IEEE Transactions on Feb 2005;20(no.1):96–101.
[25] Sutton RS, Barto AG. Reinforcement learning: an introduction. Cambridge, MA: MIT Press; 1998.
[26] Independent System Operator (ISO) New England. Hourly zonal information, available at: http://www.iso-ne.com/markets/hstdata/znl_info.